TECHNIQUES FOR CALCULATING LIMITS

1) Substitution

Calculate $\lim_{x\to 3} \frac{1+x}{1-x}$.

- When substituting 3 into x, we get the value $\frac{1+3}{1-3} = \frac{4}{-2} = -2$.
- Therefore $\lim_{x\to 3} \frac{1+x}{1-x} = -2$.

2) Simplify algebraically before substitution

a) Calculate $\lim_{x\to 1} \frac{x^2-1}{x-1}$.

- When substituting 1 into x, we get the value $\frac{1-1}{1-1} = \frac{0}{0}$ which is undefined.
- Evaluate the limit again by factorising and simplifying before substitution.
- $\lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x\to 1} x + 1$.
- Now when substituting 1 into x we get the value 2.
- Therefore $\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$.

b) Calculate $\lim_{x\to -1} \frac{1-x^2}{x^2-2x-3}$.

- Direct substitution: $\frac{1-1}{1+2-3} = \frac{0}{0}$. Evaluate again.
- $\lim_{x\to -1} \frac{1-x^2}{x^2-2x-3} = \lim_{x\to -1} \frac{(1-x)(1+x)}{(x-3)(x+1)} = \lim_{x\to -1} \frac{1-x}{x-3}$.
- Substitute x = -1: $\frac{1+1}{-1-3} = \frac{2}{-4} = -0.5$
- Therefore $\lim_{x\to -1} \frac{1-x^2}{x^2-2x-3} = -0.5$.

When substituting values into x, the general guide to determining the limit is:

Result when substituting	Conclusion
sensible, defined value	the value is the limit
$\frac{number\neq 0}{0}$ e.g. $\frac{5}{0}$	undefined, no limit
$\frac{0}{number\neq 0}$ e.g. $\frac{0}{5}$	limit is 0
$\frac{0}{0}$	factorise, simplify, substitute again

3) For the case of $x \to 0$

If substitution, or factorising before substitution, does not work -> Substitute a small number into x, and evaluate on a calculator.

If substituting -0.0001 into x produces a value different to 1, then no limit exists.

a) Calculate $\lim_{x \to 0} \frac{\sin x}{x}$.

- When substituting 0 into x, we get the value $\frac{0}{0}$.
- Function cannot be factorised.
- Substitute a small number like 0.0001 into function: $\frac{\sin 0.0001}{0.0001}$ = 0.99999.... = 1
- Therefore $\lim_{x \to 0} \frac{\sin x}{x}$ = 1.

b) Calculate $\lim_{x \to 0} x^x$.

- Direct substitution: 0^0 is undefined.
- Function cannot be factorised.
- Substitute a small number like 0.0001 into function: $0.0001^{0.0001}$ = 0.999.... = 1
- Therefore $\lim_{x \to 0} x^x$ = 1.

4) For the case of $x \to \infty$

Either:

Divide each term in the numerator and denominator by x.

The rule $\lim_{x \to \infty} \frac{a}{x} = 0$, where a is a constant, will be useful.

a) Calculate $\lim_{x \to \infty} \frac{x+1}{x}$.

- Divide all terms by x: $\frac{\frac{x}{x}+\frac{1}{x}}{\frac{x}{x}} = \frac{1+\frac{1}{x}}{1} = 1 + \frac{1}{x}$
- $\lim_{x \to \infty} \frac{x+1}{x}$ = $\lim_{x \to \infty} 1 + \frac{1}{x}$ = 1 + 0 = 1.
- Therefore $\lim_{x \to \infty} \frac{x+1}{x}$ = 1.

b) Calculate $\lim_{x \to \infty} \frac{2x-1}{x+2}$.

- Divide all terms by x: $\frac{\frac{2x}{x}-\frac{1}{x}}{\frac{x}{x}+\frac{2}{x}} = \frac{2-\frac{1}{x}}{1+\frac{2}{x}}$
- $\lim_{x \to \infty} \frac{2x-1}{x+2}$ = $\lim_{x \to \infty} \frac{2-\frac{1}{x}}{1+\frac{2}{x}} = \frac{2-0}{1+0} = \frac{2}{1} = 2$.
- Therefore $\lim_{x \to \infty} \frac{2x-1}{x+2}$ = 2.

Or:

Substitute a large number into x, and evaluate on a calculator.

a) Calculate $\lim_{x\to\infty}\frac{x+1}{x}$.

- Substitute a large number like 10000 into function: $\frac{10000+1}{10000}=\frac{10001}{10000}=1.0001=1.$
- Therefore $\lim_{x\to\infty}\frac{x+1}{x}$ = 1.

b) Calculate $\lim_{x\to\infty}\frac{2x-1}{x+2}$.

- Substitute a large number like 10000 into function: $\frac{20000-1}{10000+2}=\frac{19999}{10002}=1.9995=2.$
- Therefore $\lim_{x\to\infty}\frac{2x-1}{x+2}$ = 2.

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